Example-1
P-75

$$u(x,y) = \frac{A(x^2-y^2)}{(x^2+y^2)^2}$$

$$v(x,y) = \frac{2Axy}{(x^2+y^2)^2}$$

$$w = 0$$

since there is no external force have

$$X = Y = Z = 0$$

$$W = 0$$

$$\Rightarrow -\frac{1}{\rho}\frac{\partial P}{\partial z} = 0$$

$$\Rightarrow \quad P = P(x,y) \text{ Independent of } z$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$= -\frac{1}{\rho}\frac{\partial P}{\partial z}$$

$$\frac{\partial u}{\partial x} = A\,\frac{2x(x^2+y^2)^2 - (x^2-y^2)\cdot 2(x^2+y^2)2x}{(x^2+y^2)^4}$$

$$= A\,\frac{2x(x^2+y^2) - (x^2-y^2)4x}{(x^2+y^2)^3}$$

$$= 2Ax\,\frac{(3y^2-x^2)}{(x^2+y^2)^3}$$

Similarly

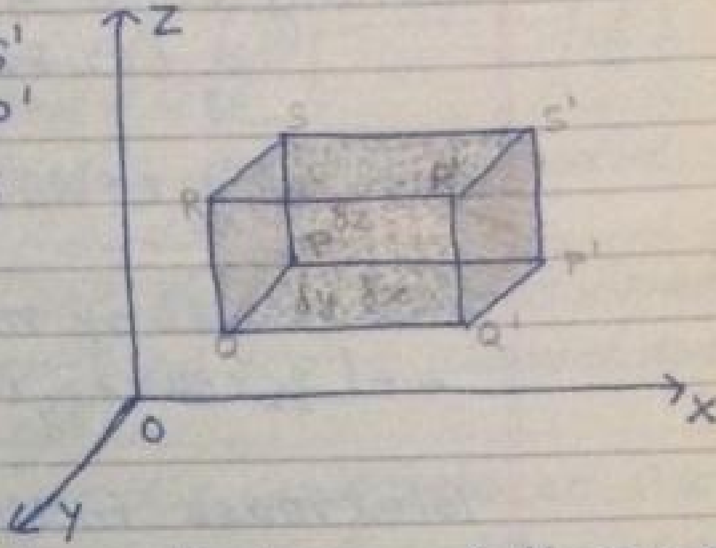
$$\frac{\partial u}{\partial y} = -2Ay\,\frac{(3x^2-y^2)}{(x^2+y^2)^3}$$

$$\frac{\partial v}{\partial x} = 2Ay\,\frac{(y^2-3x^2)}{(x^2+y^2)^3}$$

$$\frac{\partial v}{\partial y} = 2Ax\,\frac{(x^2-3y^2)}{(x^2+y^2)^3}$$

29 P-34 Equation of Continuity (cartesian cordinate)

PQRS — P'Q'R'S'
RR'S'S — QQ'PP'
RR'Q'Q — PP'S'S



Total accumulation = Express of the mass that flows in the Parallelopiped out the mass that flows out in time δt

Mass of the fluid that Passes in through the face PQRS

$= (\rho\, \delta y\, \delta z) u$ along x axis/u.m.

$= f(x, y, z)$

Mass of the fluid that passes out through the face P'Q'R'S'

$= f(x+\delta x, y, z)$ along x axis Per unit mass

Excess of the mass that flows in through the face PQRS out the mass that flows out through the face P'Q'R'S'

$= f(x, y, z) - f(x+\delta x, y, z)$

$= f(x, y, z) - \{ f(x, y, z) + \delta x \cdot \frac{\partial}{\partial x} f(x, y, z) + \}$

$= -\delta x \frac{\partial}{\partial x} f(x, y, z)$

$= -\delta x \frac{\partial}{\partial x} (\rho u\, \delta y\, \delta z)$

$= -\frac{\partial}{\partial x} (\rho u)\, \delta x\, \delta y\, \delta z)$ along x-axis

Let $F(r,t)$ denote some fluid Property of the element of $F(r+dr, t+dt)$ denote the fluid Property at the Point Q.

The rate of change of the fluid element is given as

$$\frac{DF}{Dt} = \lim_{\delta t \to 0} \frac{\delta F}{\delta t} = \lim_{\delta t \to 0} \frac{F(r+\delta r, t+\delta t) - F(r,t)}{\delta t}$$

By Taylor's Expansion

$$F(r+q\delta t, t+\delta t) = F(r,t) + \frac{\partial F}{\partial t}\delta t + \frac{\partial^2 F}{\partial t^2}(\delta t)^2 + \cdots$$

$$+ \frac{\partial F}{\partial s}(q\delta t) + \frac{\partial^2 F}{\partial s^2}(q\delta t)^2 + \cdots$$

$$\frac{DF}{Dt} = \lim_{\delta t \to 0} \frac{\partial F}{\partial t} + \frac{\partial F}{\partial s} q$$

This can we write

$$\boxed{\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \frac{\partial}{\partial s} q}$$

Dt = material derivative

∂t = Local derivative

∂s = convective derivative

This can be expressed are

(1) As a change due to Local Variation with time of the fluid Property at a given Position.

(2) due to a change of Position at a Prescribed time

Let r be the radius of the cavity and v be the velocity at any time t. then

$$r' = r \ , \ v' = v \ , \ P = 0$$

$$-\frac{f'(t)}{r} + \frac{1}{2}v^2 = \frac{2\mu}{r^{1/2}}$$

Again $\quad f(t) = r^2 v$

Diff. we have

$$f'(t) = 2r\frac{\partial r}{\partial t}v + r^2\frac{\partial v}{\partial t}$$

$$= 2rv^2 + r^2\frac{\partial v}{\partial r}\frac{\partial r}{\partial t}$$

$$= 2rv^2 + r^2 v\frac{\partial v}{\partial r}$$

or $\quad -\left(2v^2 + rv\frac{dv}{dr}\right) + \frac{1}{2}v^2 = \frac{2\mu}{r^{1/2}}$

or $\quad 2v^2 + rv\frac{dv}{dr} + \frac{1}{2}v^2 = -\frac{2\mu}{r^{1/2}}$

or $\quad rv\frac{dv}{dr} + \frac{3}{2}v^2 = -\frac{2\mu}{r^{1/2}}$

Multiplying both sides with $2r^2 dr$ and Integrating, we get

$$2r^3 v\,dv + 3r^2v^2\,dr = -4\mu r^{3/2}\,dr$$

$$r^3v^2 = -\frac{8\mu}{5}r^{5/2} + B$$

Now $\quad r = c, \ v = 0$

$$\Rightarrow B = \frac{8\mu}{5}c^{5/2}$$

$$r^3 v^2 = -\frac{8\mu}{5} r^{5/2} + \frac{8\mu}{5} c^{5/2}$$

$$= \frac{8\mu}{5} (c^{5/2} - r^{5/2})$$

$$v^2 = \frac{8\mu}{5} \frac{(c^{5/2} - r^{5/2})}{r^3}$$

$$v = \frac{dr}{dt} = \pm \sqrt{\frac{8\mu}{5}} \cdot \sqrt{\frac{(c^{5/2} - r^{5/2})}{r^3}}$$

$$\int_0^T dt = \pm \frac{5}{8\mu} \int \frac{r^{3/2}}{(c^{5/2} - r^{5/2})} dr$$

$-ve$ sign is taken between time increases distance r decreases.

Let $r^{5/2} = c^{5/2} \sin^2\theta$

$$\frac{5}{2} r^{3/2} dr = 2c^{5/2} \sin\theta \cos\theta \, d\theta$$

or $$t = \frac{4}{5} \sqrt{\left(\frac{5}{8\mu}\right)} \int_0^{\pi/2} \frac{c^{5/2} \sin\theta \cos\theta}{c^{5/4} \cos\theta} d\theta$$

or $$t = \frac{4}{5} c^{5/4} \sqrt{\left(\frac{5}{8\mu}\right)} \int_0^{\pi/2} \sin\theta \, d\theta$$

$$t = \sqrt{\left(\frac{2}{5\mu}\right)} c^{5/4} \quad \text{Ans.}$$

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